- Let's consider a rigid link that is rotating about pt. O_2

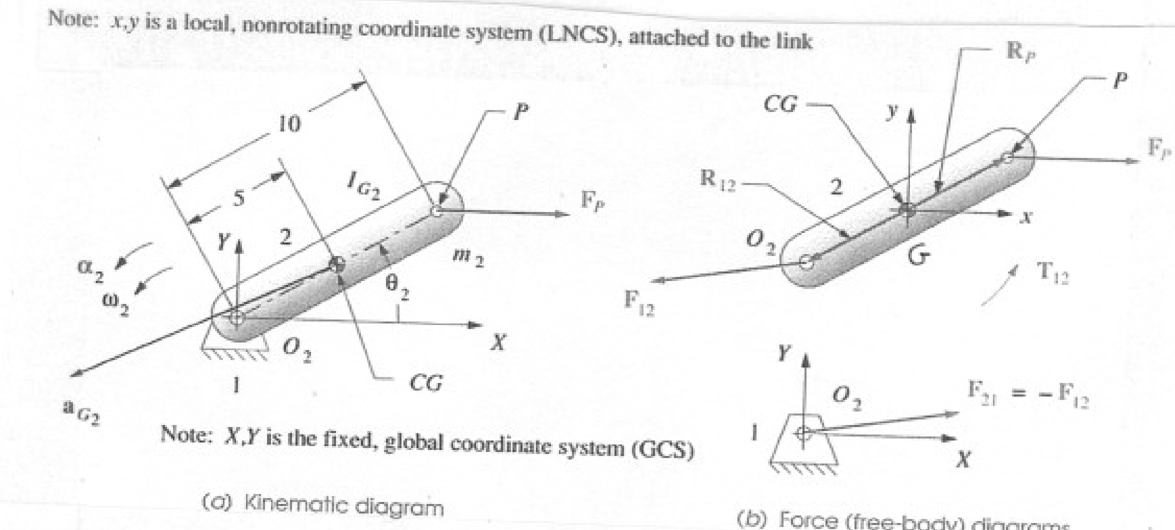


FIGURE 11-1

Dynamic force analysis of a single link in pure rotation

In order to solve the Kinetostatic force analysis we must already know the kinematics. (angular accelerations of all rotating members and the linear accelerations of the CGs of all moving members) We also need to know the mass, moment of inertia wrt. each CG and all the external forces and torques

Known: α_2, a_{G2}, m_2, I_{G2}, F_P

Unknowns: F_{12} = Force of link 1 acting on link 2

T_{12} = Torque of link 1 acting on link 2

Note: F_P and F_{12} have x and y components (F_{Px}, F_{Py}, F_{12x}, F_{12y}) and T_{12} is likely due to friction or an input

- The first step in solving this problem is to create a local coordinate system on each moving member at its CG and to draw the appropriate Free Body Diagram (FBD)

- The point of application of the forces are defined by the position vectors R_{12} and R_P with respect to the local coordinate system (see FBD).

- Now we can write Newton's Law

$$\sum \vec{F} = \vec{F}_P + \vec{F}_{12} = m_2 \vec{a}_G$$

$$\sum T = \vec{T}_{12} + (\vec{R}_{12} \times \vec{F}_{12}) + (\vec{R}_P \times \vec{F}_P) = I_G \vec{\alpha}$$

- and we can break the problem up into x and y components

$$+\rightarrow \quad \sum F_x = F_{Px} + F_{12x} = m_2 a_{Gx}$$

$$+\uparrow \quad \sum F_y = F_{Py} + F_{12y} = m_2 a_{Gy}$$

$$\circlearrowleft\!+ \quad \sum T = T_{12} + (R_{12x} F_{12y} - R_{12y} F_{12x}) + (R_{Px} F_{Py} - R_{Py} F_{Px}) = I_G \alpha$$

OR

$$\circlearrowleft\!+ \quad \sum T = T_{12} + (F_{12y}\, \overline{O_2G} \cos\theta_2 - F_{12x} \overline{O_2G} \sin\theta_2) + (F_{Py}\, \overline{PG} \cos\theta_2 - F_{Px}\, \overline{PG} \sin\theta_2) = I_G \alpha$$

We can put these equations into Matrix Form to solve for the unknown Forces and torques

$$\underset{\text{Geometry}}{[A]}\ \underset{\text{Unknowns}}{[B]} = \underset{\text{Dynamic Information}}{[C]}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\overline{O_2G}\sin\theta_2 & \overline{O_2G}\cos\theta_2 & 1 \end{bmatrix} \begin{bmatrix} F_{12x} \\ F_{12y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{Gx} - F_{Px} \\ m_2 a_{Gy} - F_{Py} \\ I_G \alpha + F_{Px} \overline{PG} \sin\theta_2 - F_{Py} \overline{PG} \cos\theta_2 \end{bmatrix}$$

$$[B] = [A]^{-1}[C]$$

Example

For the linkage shown on page 16-1

length = $\overline{O_2P} = 10''$, $\overline{O_2G} = 5''$, $\overline{PG} = 5''$

$I_G = 0.08 \text{ lb in sec}^2$, $W = 4 \text{lbs}$

$\theta_2 = 30°$, $\omega_2 = 20 \text{ r/s}$, $\alpha_2 = 15 \text{ r/s}^2$, $a_{G2} = 166.75 \text{ ft/s}^2 \angle 208°$

$F_P = 40 \text{lbs} \angle 0°$

Find F_{12} and T_{12}

Solution

- First we need to convert the weight and dimensions to proper units.

(mks) $F = ma$ $\text{Newton} = (\text{Kilogram})\left(\dfrac{\text{meter}}{\text{second}^2}\right)$

(fps) $F = ma$ $\text{pound force} = (\text{slug})\left(\dfrac{\text{ft}}{\text{second}^2}\right)$

If you use either of the expressions above you don't have to worry about g_c $\left(F = \dfrac{ma}{g_c}\right)$ when using pound mass (lb_m)

1 blob = 12 slug = 386 lbm 1 slug = 32.2 lbm

↑ used in the inch-pound-second system

$$W = mg \Rightarrow m = \frac{W}{g} = \frac{4 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.12422 \text{ slug}$$

- Now we need to calculate the x and y components

$a_G = 166.75 \dfrac{\text{ft}}{\text{s}^2} \angle 208°$ → use Euler's identity

$a_{Gx} = -147.23 \dfrac{\text{ft}}{\text{s}^2}$ $a_{Gy} = -78.28 \text{ ft/s}^2$

$F_{px} = 40 \text{ lbs} \qquad F_{py} = 0$

$I_G = 0.08 \text{ lb in } s^2 \cdot \frac{ft.}{12 in.} = 6.667 \times 10^{-3} \text{ lb ft } s^2$

Now substitute the values into the matrix Equation $A = BC$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\left(\frac{5}{12}\right)\sin(30^\circ) & \frac{5}{12}\cos(30^\circ) & 1 \end{bmatrix} \begin{bmatrix} F_{12x} \\ F_{12y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} 0.12422(-147.23) - 40 \\ 0.12422(-78.28) - 0 \\ (6.667\times10^{-3})(15) + 40\left(\frac{5}{12}\right)\sin(30^\circ) \end{bmatrix}$$

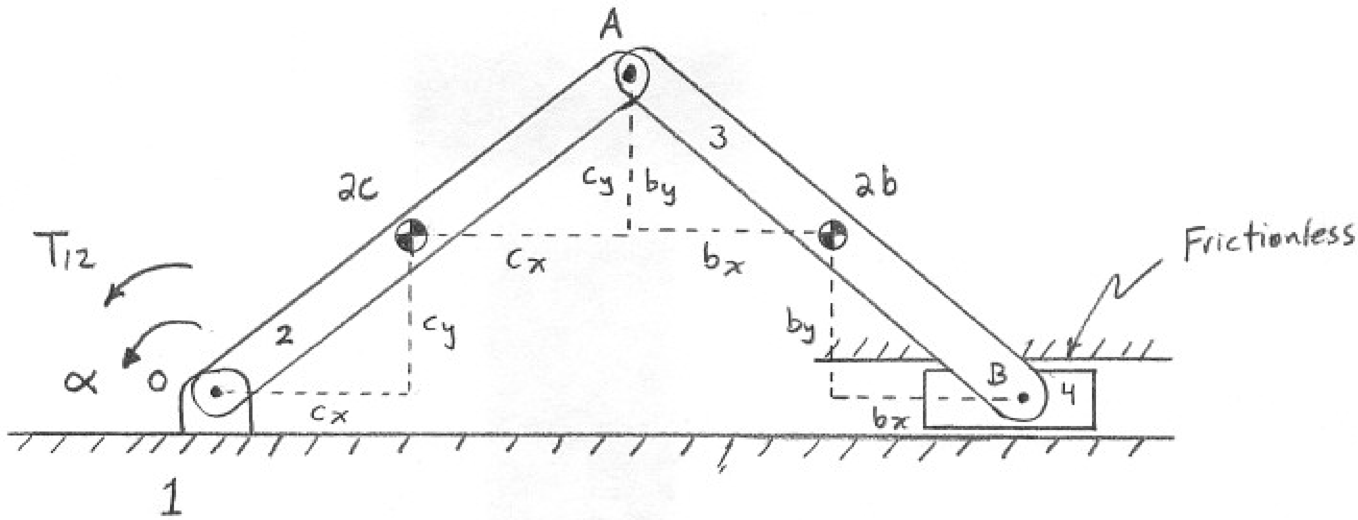
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -0.2083 & 0.3608 & 1 \end{bmatrix} \begin{bmatrix} F_{12x} \\ F_{12y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} -58.3 \\ -9.725 \\ 8.433 \end{bmatrix}$$

$$\begin{bmatrix} F_{12x} \\ F_{12y} \\ T_{12} \end{bmatrix} = A^{-1}C = \begin{bmatrix} -58.3 \text{ lbs} \\ -9.73 \text{ lbs} \\ -0.201 \text{ lbft} \end{bmatrix}$$

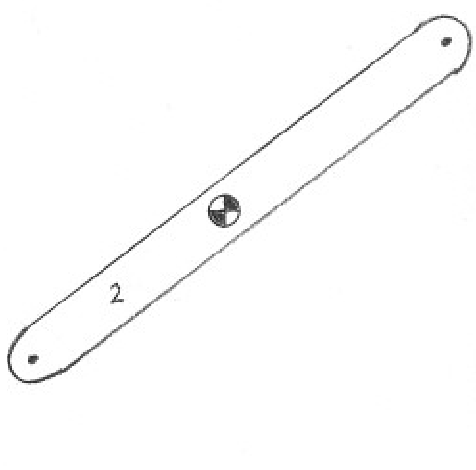
$\boxed{T_{12} = -2.42 \text{ in lb}}$

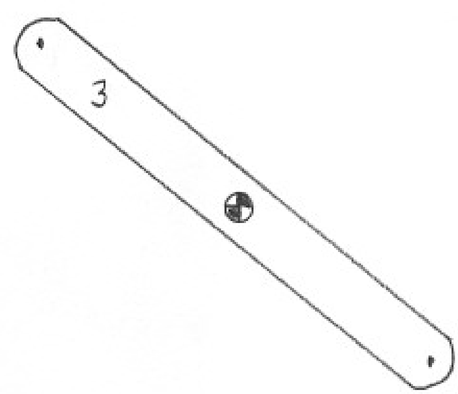
$\boxed{F_{12} = 59.1 \text{ lbs} \angle 189.5^\circ}$

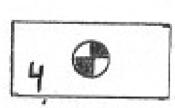
Consider the following slider - crank mechanism. We can assume that the slider is frictionless and the friction in pin O2 is negligible compared to torque input T_{12}



Draw the FBDs and determine the unknowns







Unknowns :